SET B

2019

Roll No.

This question paper contains 2 printed pages

(23)

Unique paper code : 235304

Name of the course : B. Sc. (Hons) Mathematics

Name of the paper : MAHT 303 -- Algebra-II

Semester : III

Duration : 3 Hours

Maximum marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of the question paper.
2. Attempt any *two* parts from each question.
3. **All** questions are compulsory.

1. (a) Prove that $Z_n = \{0, 1, \ldots, n-1\}$ is a group under the addition modulo n.

(b) Let H be a nonempty subset of a group G. Then prove that if $ab^{-1} \in H$ for all $a, b \in H$, then H is a subgroup of G.

(c) Define Center $Z(G)$ of a group G. Also, prove that $Z(G)$ is a subgroup of G.

(6, 6, 2+4)

2. (a) In a group G, prove the following :

(i) the identity element of G is unique,

(ii) the cancellation laws hold in G.

(b) Let a be an element of a group G such that $|a|$, the order of a, is finite. If $a^k = e$, then prove that $|a|$ divides k.

(c) Define a cyclic group. Let a and b be elements in a group G with $|a| = m$, $|b| = n$. If $(m, n) = 1$, then prove that $< a > \cap < b > = \{e\}$.

(3+3, 6, 2+4)

3. (a) Let $\alpha = (a_1, a_2 \ldots a_m)$ and $\beta = (b_1, b_2 \ldots b_k)$ be disjoint cycles. Then prove that $\alpha\beta = \beta\alpha$.

(b) Prove that the order of a permutation of a finite set written in disjoint cycles form is the LCM of the lengths of the cycles.

(c) State and prove Orbit Stablizer Theorem.

(6, 6, 2+4)

4. (a) Let H be a subgroup of G and let a and b be elements in G. Then, prove that

(i) $|aH| = |bH|$,

(ii) $aH = Ha$ if and only if $H = aHa^{-1}$.

(b) Prove that every subgroup of an Abelian group is normal. Is the converse true? Justify.

(c) Let N be a normal subgroup of a group G with order 2. Then prove that N is contained in the center Z(G) of G.

(6.5, 3.5+3, 6.5)

5. (a) Prove that the alternating group A_n is a normal subgroup of S_n.

(b) Let G be a finite Abelian group and let p be a prime that divides the order of G. Prove that G has an element of order p.

(c) Find the factor group $Z/4Z$.

(6.5, 6.5, 6.5)

6. (a) Find all homomorphisms from Z_{12} to Z_{30}.

(b) Let θ be a group homomorphism from G to K. Let H be a subgroup of G. Prove that

(i) $\theta(H)$ is a subgroup of K.

(ii) If H is Abelian, then $\theta(H)$ is also Abelian.

(c) Let θ be a group homomorphism from G to K and let $g \in G$. If $\theta(g) = g'$, then prove that $\theta^{-1}(g') = gKer\,\theta$.

(6.5, 6.5, 6.5)